

Discharging **POs** of original m0: Invariant Preservation

ML_out/inv0_1/INV

$d \in \mathbb{N}$
 $n \in \mathbb{N}$
 $n \leq d$
 \vdash
 $n + 1 \in \mathbb{N}$

ML_in/inv0_1/INV

$d \in \mathbb{N}$
 $n \in \mathbb{N}$
 $n \leq d$
 \vdash
 $n - 1 \in \mathbb{N}$

ML_out/inv0_2/INV

$d \in \mathbb{N}$
 $n \in \mathbb{N}$
 $n \leq d$
 \vdash
 $n + 1 \leq d$

ML_in/inv0_2/INV

$d \in \mathbb{N}$
 $n \in \mathbb{N}$
 $n \leq d$
 \vdash
 $n - 1 \leq d$

$H \vdash P$	OR_R1
$H1 \vdash G$	MON
$n \leq m \vdash n - 1 < m$	DEC
$n \in \mathbb{N} \vdash n + 1 \in \mathbb{N}$	P2

PO/VC Rule of Invariant Preservation: Revised M0

constants: d

variables: n

axioms:
 $\text{axm0_1} : d \in \mathbb{N}$

invariants:
 $\text{inv0_1} : n \in \mathbb{N}$
 $\text{inv0_2} : n \leq d$

ML_out
begin
 $n := n + 1$
end

ML_in
begin
 $n := n - 1$
end

$A(c)$
 $I(c, v)$
 $G(c, v)$

\vdash

$I_i(c, E(c, v))$

Q. How many PO/VC rules for model m0?

Discharging POs of revised m0: Invariant Preservation

ML_out/inv0_1/INV

$d \in \mathbb{N}$
 $n \in \mathbb{N}$
 $n \leq d$
 $n < d$
 \vdash
 $n + 1 \in \mathbb{N}$

ML_in/inv0_1/INV

$d \in \mathbb{N}$
 $n \in \mathbb{N}$
 $n \leq d$
 $n > 0$
 \vdash
 $n - 1 \in \mathbb{N}$

ML_out/inv0_2/INV

$d \in \mathbb{N}$
 $n \in \mathbb{N}$
 $n \leq d$
 $n < d$
 \vdash
 $n + 1 \leq d$

ML_in/inv0_2/INV

$d \in \mathbb{N}$
 $n \in \mathbb{N}$
 $n \leq d$
 $n > 0$
 \vdash
 $n - 1 \leq d$

$$\frac{H \vdash P}{H \vdash P \vee Q} \text{ OR.R1}$$

$$\frac{H_1 \vdash G}{H_1, H_2 \vdash G} \text{ MON}$$

$$\frac{}{n \leq m \vdash n - 1 < m} \text{ DEC}$$

$$\frac{}{n < m \vdash n + 1 \leq m} \text{ INC}$$

$$\frac{}{n \in \mathbb{N} \vdash n + 1 \in \mathbb{N}} \text{ P2}$$

$$\frac{}{0 < n \vdash n - 1 \in \mathbb{N}} \text{ P2'}$$